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Ghost free dual vector theories in $2+1$ dimensions

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Abstract: We explore here the issue of duality versus spectrum equivalence in dual theories generated through the master action approach. Specifically we examine a generalized self-dual (GSD) model where a Maxwell term is added to the self-dual model. A gauge embedding procedure applied to the GSD model leads to a Maxwell-Chern-Simons (MCS) theory with higher derivatives. We show here that the latter contains a ghost mode contrary to the original GSD model. By figuring out the origin of the ghost we are able to suggest a new master action which interpolates between the local GSD model and a nonlocal MCS model. Those models share the same spectrum and are ghost free. Furthermore, there is a dual map between both theories at classical level which survives quantum correlation functions up to contact terms. The remarks made here may be relevant for other applications of the master action approach.

Keywords: Gauge Symmetry, Duality in Gauge Field Theories, Chern-Simons Theories.

Contents

1. Introduction

The bosonization program in two dimensions is a good example of the power of dual descriptions of the same theory [1, 2], see also [3] and references therein. Although much less progress has been done in this area in higher dimensions, some nonperturbative features like confinement can also be revealed with the help of duality as in [4]. Here we are interested in the intermediate case of $2+1$ dimensions. By means of a master action it was shown in [5] that the gauge invariant sector of the Maxwell Chern-Simons (MCS) theory is on shell equivalent to the self-dual (SD) theory of [6] with the dual map $f_\mu \leftrightarrow \frac{\epsilon_{\mu\nu\alpha}\partial^\nu A^\alpha}{m}$. This type of duality is intimately related to the bosonization program in $d = 2+1$ [7−12].

The master action of [5] has been generalized to include matter fields in [13−15]. Although very useful specially at quantum level in order to provide equivalence of correlation functions, the master action is not derived from first principles and it is justified *a posteriori*. A systematic derivation of dual gauge theories would certainly be welcome. In [14] a Noether gauge embedding procedure was suggested and latter, see [16−18], it was applied on a variety of examples and dimensions. However, it has been pointed out in [19] that this procedure may lead to ghosts in the dual gauge theory which has been explicitly verified in a dual model to a four dimensional Lorentz violating electrodynamics [20]. Here we show that this is also the case of a gauge theory, suggested in [18], which is dual to a generalized self-dual model. An understanding of this issue from the point of view of master actions is the aim of this work. In the particular case analyzed in [18] we propose an alternative master action to avoid the presence of ghosts but our interpretation is rather general and can be used to suggest master actions in other cases. In order to be able to make a non-perturbative analysis of the spectrum we choose quadratic theories. We pick up the case of vector theories in $d = 2+1$ since the issue of generating masses without breaking gauge invariance is very important in particle physics and that seems to be behind the difficulties in generating local dual vector theories with more than one massive mode in $d = 2+1$. In particular, we start in the next section from a rather general quadratic non-gauge theory for a vector field in $d = 2+1$ with two massive modes and calculate the residues of the propagator around those poles in order to verify the presence of ghosts. In the same section we apply the so called Noether embedding procedure to generate the corresponding dual gauge theory. In section 3 we show how the use of master actions may lead to dual theories with different spectra and we learn how to modify it in order to avoid a spectrum mismatch in the dual gauge theory. In the last section we draw some conclusions.

2. The GSD model and its higher derivative dual

Allowing arbitrary functions of the D'Alambertian $a_i = a_i(\Box)$, the lagrangian below includes a rather general set of Poincaré covariant quadratic theories for an abelian vector field in $d = 2+1$, see also [21, 18],

$$\mathcal{L}_{GSD} = a_0 f^\mu f_\mu + a_1 \epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma + \frac{a_2}{2} F_{\mu\nu}(f) F^{\mu\nu}(f) \tag{2.1}$$

We use $g_{\mu\nu} = (+,-,-)$ and assume in this work that a_i are constants but later on we will make a comment on the general case. Minimizing $\mathcal{L}_{GSD}$ we can write its equations of motion in a self-dual form where $\tilde{f}$ defined below stands for the dual field:

$$f_\mu = \frac{a_1}{a_0} E_{\mu\nu} f^\nu + \frac{a_2}{a_0} \Box\, \theta_{\mu\nu} f^\nu \equiv \tilde{f}_\mu \tag{2.2}$$

We have introduced $E_{\mu\nu} = \epsilon_{\mu\nu\gamma}\partial^\gamma$ and $\Box\, \theta_{\mu\nu} = g_{\mu\nu} - \partial_\mu \partial_\nu$. From (2.2) we get $\partial_\mu f^\mu = 0$. Since $E_{\mu\nu} E^{\nu\gamma} = -\Box\, \theta_\mu^\gamma$ and both a_0 and a_1 are presumably non-vanishing, it is easy to derive from (2.2):

$$\left[(a_0 - a_2 \Box)^2 + a_1^2 \Box \right] f^\mu = a_2^2 \left(\Box + m_+^2 \right) \left(\Box + m_-^2 \right) f^\mu = 0 \tag{2.3}$$

with

$$2 m_\pm^2 = b^2 - 2a \pm \sqrt{(b^2 - 2a)^2 - 4a^2} \tag{2.4}$$

and $a = a_0/a_2$, $b = a_1/a_2$. The masses are real for $a < 0$ or $b^2 > 4a$ if $a > 0$. An interesting observation is that we can not have $(a_0 - a_2 \Box) f^\mu = 0$, since from (2.3) we see that this would lead to $\Box f^\mu = 0$ and this two equations together would contradict our hypothesis $a_0 \neq 0$. We can read off the propagator for the generalized self-dual field from (2.1). In the momentum space we have:

$$\langle f_\alpha(k) f_\beta(-k) \rangle_{GSD} = \frac{g_{\alpha\beta} - \theta_{\alpha\beta}}{a_0} + \frac{(a_0 + k^2 a_2)}{(a_0 + k^2 a_2)^2 - a_1^2 k^2} \theta_{\alpha\beta} + \frac{i\, a_1 E_{\alpha\beta}}{(a_0 + k^2 a_2)^2 - a_1^2 k^2} \tag{2.5}$$

As expected from (2.3) there are two simple poles at $k^2 = m_\pm^2$ which become a double pole only at the special point $a_1^2 = 4 a_0 a_2$. In order to make sure that we are free of ghosts we have to show in general, see e.g. [20−22], that the imaginary part of the residue of the propagator at each pole is positive when saturated with conserved currents, that is $Im \left(Res(J_\alpha \langle f^\alpha(k) f^\beta(-k) \rangle J_\beta^*) \right) > 0$ with $k_\mu J^\mu = 0 = k^\mu J_\mu^*$. In $d = 2+1$ the propagator of a vector field in a Poincarè covariant theory will have the general Lorentz structure $A(k^2)(g_{\mu\nu} - \theta_{\mu\nu}) + B(k^2)\theta_{\mu\nu} + C(k^2)E_{\mu\nu}$. In practice[1], this is equivalent to require $\lim_{k^2\to m^2}(k^2 - m^2)B(k^2) < 0$. Applying this requirement at $k^2 = m_+^2$ and $k^2 = m_-^2$ respectively we have:

$$\frac{a + m_+^2}{a_2(m_+^2 - m_-^2)} = \frac{b^2 + \sqrt{b^2(b^2 - 4a)}}{2a_2(m_+^2 - m_-^2)} < 0 \tag{2.6}$$

$$\frac{a + m_-^2}{a_2(m_-^2 - m_+^2)} = -\frac{b^2 - \sqrt{b^2(b^2 - 4a)}}{2a_2(m_+^2 - m_-^2)} < 0 \tag{2.7}$$

Since $m_+^2 > m_-^2$ we are free of ghosts if $a_0 > 0$ and $a_2 < 0$, in agreement with [21]. Those are in fact usual conditions on the mass term of the self-dual model and on the coefficient of the Maxwell term in QED in $d = 2+1$ respectively. Notice that we get rid automatically of the double pole under such circumstances.

In [18] the gauge theory dual to $\mathcal{L}_{GSD}$ was derived via a Noether gauge embedding procedure which in summary works as follows. Since the equations of motion for the the self-dual field come from $K^\mu = 2\left(a_0 f^\mu - a_1 E^{\mu\nu} f_\nu - a_2 \Box\, \theta^{\mu\nu} f_\nu\right) = 0$, and under a local U(1) transformation $\delta^\phi f_\mu = \partial_\mu \phi$ we have $\delta^\phi K_\mu = 2a_0 \delta^\phi f_\mu$. Thus, if we define $\mathcal{L}_{HD-MCS} = \mathcal{L}_{GSD} - K_\alpha K^\alpha / 4a_0$ it follows that $\delta^\phi \mathcal{L}_{HD-MCS} = 0$ and we have a gauge invariant theory. By minimizing this theory $\delta\mathcal{L}_{HD-MCS} = K^\mu \delta f_\mu - K^\mu \delta K_\mu / 2a_0 = 0$ it is clear that the equations of motion $K_\mu = 0$ of $\mathcal{L}_{GSD}$ lead to $\delta\mathcal{L}_{HD-MCS} = 0$ and therefore are embedded in the equations of motion of the new gauge theory. We stress that such embedding does not guarantee equivalence of the equations of motion of both theories, not even in the gauge invariant sector of $\mathcal{L}_{HD-MCS}$. Applying this embedding explicitly, renaming the field $f_\mu \to A_\mu$, one has the higher derivative theory [18]:

$$\begin{aligned} \mathcal{L}_{HD-MCS} &= \mathcal{L}_{GSD} - \frac{K_\alpha K^\alpha}{4a_0} \\ &= A^\mu a_1 \left(1 - \frac{2a_2\Box}{a_0}\right) \epsilon_{\mu\nu\gamma}\partial^\gamma A^\nu - \frac{1}{2} F_{\mu\nu}(A) \left(\frac{a_1^2}{a_0} - \frac{a_2^2\Box}{a_0} + a_2\right) F^{\mu\nu}(A) \end{aligned} \tag{2.8}$$

The equations of motion of the higher derivative model (2.8) can be written as

$$a_1 \left(1 - \frac{2a_2\Box}{a_0}\right) \epsilon_{\mu\nu\gamma}\partial^\gamma A^\nu + \Box \left(\frac{a_1^2}{a_0} - \frac{a_2^2\Box}{a_0} + a_2\right) \theta_{\mu\nu} A^\nu = a_0 \left(\tilde{A}_\mu - \tilde{\tilde{A}}_\mu\right) = 0 \quad , \tag{2.9}$$

which makes evident, see (2.2), the duality at classical level with the GSD model upon replacing f_μ by $\tilde{A}_\mu$. From (2.9) we deduce the analogous of formula (2.3):

$$\left(a_1^2 + a_2^2 \Box\right)\left(\Box + m_+^2\right)\left(\Box + m_-^2\right) \epsilon_{\mu\nu\gamma}\partial^\gamma A^\nu = 0 \tag{2.10}$$

The expression (2.10) shows that we have new classical solutions in the gauge invariant sector $\left(a_1^2 + a_2^2\Box\right)\epsilon_{\mu\nu\gamma}\partial^\gamma A^\nu = 0$ which were not present in the GSD model. The new

[1]By choosing a convenient frame for a massive pole $k = (k, 0, 0)$, which implies $J_\mu = (0, J_1, J_2)$, and using the general formula for a propagator in $d = 2+1$ one can show that $Im\left(Res(J_\alpha \left\langle f^\alpha(k) f^\beta(-k)\right\rangle J_\beta^*)\right) = -SJ\bar{J} = -S(J_1 + iJ_2)(J_1^* - iJ_2^*)$ where for every massive single pole $S = \lim_{k^2\to m^2}(k^2 - m^2)B(k^2)$.

solutions will correspond to a new pole in the propagator. After introducing a gauge fixing term with parameter λ we have the propagator:

$$\langle A_\alpha(k)A_\beta(-k)\rangle_{HD-MCS} = \frac{g_{\alpha\beta}-\theta_{\alpha\beta}}{\lambda k^2} + B(k^2)\theta_{\alpha\beta} + \frac{ia_0a_1(a_0+2a_2k^2)E_{\alpha\beta}}{k^2(a_1^2-k^2a_2^2)\left[(a_0+k^2a_2)^2-a_1^2k^2\right]} \tag{2.11}$$

where, in agreement with the general predictions of [19], we have

$$B(k^2) = \frac{\left(a_0+k^2a_2\right)}{\left(a_0+k^2a_2\right)^2-a_1^2k^2} - \frac{a_2}{a_2^2k^2-a_1^2} \tag{2.12}$$

Since the first term in $B(k^2)$ is the same one appearing in (2.5) it is clear that $k^2=m_\pm^2$ are still physical poles as in the GSD model for $a_0>0$ and $a_2<0$. However, since $\lim_{k^2\to(a_1/a_2)^2}\left[k^2-(a_1/a_2)^2\right]B(k^2) = -1/a_2$ should be negative the new pole will be a ghost if we insist in $a_2<0$. On the other hand, if we drop that condition one of the poles $k^2=m_\pm^2$ will become a ghost mode. From (2.12) we see that the only possible exit might be a fine tuning of a_0,a_1,a_2 such that one of the poles $m_\pm^2$ coincides with $(a_1/a_2)^2$ and the corresponding residue has the right sign. This is only possible for $a_1^2=-(a_0a_2)/2$. In this case $m_-^2=(a_1/a_2)^2$ but $B(k^2)=-a_0/\left[a_2^2(k^2-m_-^2)(k^2-m_+^2)\right]$ which implies that one of the poles $m_\pm^2$ must be a ghost again. In conclusion, the dual gauge model $\mathcal{L}_{HD-MCS}$ obtained by the Noether embedding procedure will always contain a ghost in the spectrum as far as $a_2\neq 0$. We only recover a ghost free theory in the well known case of the SD/MCS duality [5] where $a_2=0$.

3. New master action for $a_2\neq 0$

We could not have predicted from the start that the embedding procedure would lead us to ghosts. It turns out that is much simpler to understand the appearance of ghosts from the master action approach. In [18] a master action which relates $\mathcal{L}_{GSD}$ and $\mathcal{L}_{HD-MCS}$ was suggested:

$$\begin{aligned}\mathcal{L}_{Master} &= a_0f^\mu f_\mu + a_1\epsilon_{\alpha\beta\gamma}f^\alpha\partial^\beta f^\gamma + \frac{a_2}{2}F_{\mu\nu}(f)F^{\mu\nu}(f)\\ &\quad -a_1\epsilon_{\alpha\beta\gamma}(A^\alpha-f^\alpha)\partial^\beta(A^\gamma-f^\gamma) - \frac{a_2}{2}F_{\mu\nu}(A-f)F^{\mu\nu}(A-f)\end{aligned} \tag{3.1}$$

After the translation $A_\mu\to A_\mu+f_\mu$ in (3.1), which has a trivial jacobian in the path integral, we have two decoupled theories:

$$\mathcal{L}_{Master}\to\mathcal{L}_{GSD}(f)+a_1A^\mu\epsilon_{\mu\nu\gamma}\partial^\gamma A^\nu - \frac{a_2}{2}F_{\mu\nu}(A)F^{\mu\nu}(A) \tag{3.2}$$

If, in the path integral we further integrate over the gauge field A_μ we end up with a partition function whose lagrangian density is simply $\mathcal{L}_{GSD}(f)$. On the other hand, we could have decoupled the fields through the translation $f_\mu\to f_\mu+\tilde{A}_\mu$ which gives:

$$\mathcal{L}_{Master}\to\mathcal{L}_{HD-MCS}(A)+a_0f^\mu f_\mu \tag{3.3}$$

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By gaussian integrating over f_μ we derive the partition function for $\mathcal{L}_{HD-MCS}(A)$ which proves the correctness of (3.1). Now we can understand the appearance of ghosts as follows. Since the spectrum of (3.2) and (3.3) must be the same and f_μ in (3.3) is certainly a non-propagating field, the poles of $\mathcal{L}_{GSD}(f)$ and of the MCS theory for A_μ in (3.2) must be contained altogether in $\mathcal{L}_{HD-MCS}(A)$. Indeed, the extra pole we found in (2.11) is precisely associated with the two last terms in (3.2) including the non-propagating massless pole. The above translations also explain why a master action gives rise to two theories, in this case $\mathcal{L}_{GSD}(f)$ and $\mathcal{L}_{HD-MCS}(A)$, with different spectra.

After the above discussions it becomes clear how to modify the master action given in (3.1) to avoid a spectrum mismatch. The key point is to use a topological (non-propagating) Chern-Simons term to mix the gauge and non-gauge fields:

$$\mathcal{L}^{new}_{Master} = a_0 f^\mu f_\mu + a_1 \epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma + \frac{a_2}{2} F_{\mu\nu}(f) F^{\mu\nu}(f) - a_1 \epsilon_{\alpha\beta\gamma} (A^\alpha - f^\alpha) \partial^\beta (A^\gamma - f^\gamma) \quad (3.4)$$

Performing a translation $A_\mu \to A_\mu + f_\mu$ we have a pure Chern-Simons term plus the GSD model given in (2.1):

$$\mathcal{L}^{new}_{Master} \to \mathcal{L}_{GSD} - a_1 \epsilon_{\alpha\beta\gamma} A^\alpha \partial^\beta A^\gamma \quad (3.5)$$

while a convenient translation in the f_μ field would lead to

$$\mathcal{L}^{new}_{Master} \to \mathcal{L}_{NL-MCS} + f^\mu \left(a_0 g_{\mu\nu} - a_2 \Box \theta_{\mu\nu} \right) f^\nu \quad (3.6)$$

where the new lagrangian is a kind of nonlocal MCS theory:

$$\mathcal{L}_{NL-MCS} = -a_1 \epsilon_{\mu\nu\alpha} A^\mu \partial^\nu A^\alpha - F^{\mu\nu}(A) \frac{a_1^2}{2(a_0 - a_2\Box)} F_{\mu\nu}(A) \quad (3.7)$$

Notice that for $a_2 = 0$ we recover the duality between the SD and MCS models. In order to check that the spectrum of $\mathcal{L}_{NL-MCS}$ and $\mathcal{L}_{GSD}$ are the same we write down the propagator coming from (3.7) after adding a gauge fixing term with parameter λ:

$$\langle A_\alpha(k) A_\beta(-k) \rangle_{NL-MCS} = \frac{g_{\alpha\beta} - \theta_{\alpha\beta}}{\lambda k^2} + \frac{(a_0 + k^2 a_2)}{(a_0 + k^2 a_2)^2 - a_1^2 k^2} \theta_{\alpha\beta} + \frac{i\,(a_0 + k^2 a_2)^2\, E_{\alpha\beta}}{a_1 k^2 \left[(a_0 + k^2 a_2)^2 - a_1^2 k^2 \right]} \quad (3.8)$$

Comparing with the propagator coming from the GSD model (2.5) , except for the non-physical (gauge dependent) massless pole $k^2 = 0$ which stems from the Chern-Simons term in (3.5) the theories $\mathcal{L}_{GSD}$ and $\mathcal{L}_{NL-MCS}$ share precisely the same spectrum consisting of two massive physical particles in general. Concerning the Chern-Simons pole $k^2 = 0$, the reader can easily check by choosing a frame $k_\mu = (k, k, 0)$ and saturating with conserved currents that we have $\epsilon^{\mu\nu\alpha} J_\mu J^*_\nu k_\alpha = 0$ and therefore , as noticed in [19], this massless pole has a vanishing residue and does not propagate. Thus, the conclusion is that we better mix the gauge and non-gauge fields in the master action through a non-propagating term as the topological Chern-Simons term in the specific case of $d = 2 + 1$.

Once we have checked the particle content we now verify the dual map at classical level. The new master action furnishes the equations of motion:

$$d(A - f) = 0 \quad (3.9)$$

$$f_\mu = \tilde{f}_\mu \quad (3.10)$$

The duality transformation is defined as in section 2. The equation (3.10) is the same of the GSD model. From (3.9) and (3.10) we have $A_\mu + \partial_\mu\phi = \tilde{A}_\mu$ which leads to $a_2^2\left(\Box + m_+^2\right)\left(\Box + m_-^2\right)\tilde{A}_\mu = 0$. Since the dual of a total divergence vanishes we have $\tilde{A}_\mu = \tilde{\tilde{A}}_\mu$ which guarantees the classical duality between $\mathcal{L}_{NL-MCS}$ and $\mathcal{L}_{GSD}$ using the map $f_\mu \leftrightarrow \tilde{A}_\mu$. On the other hand, from (3.1) we have:

$$\tilde{f}_\mu = \tilde{A}_\mu \tag{3.11}$$

$$f_\mu = \tilde{f}_\mu \tag{3.12}$$

They also lead to $\tilde{A}_\mu = \tilde{\tilde{A}}_\mu$. If $a_2 = 0$, $\mathcal{L}_{Master}^{new}$ and $\mathcal{L}_{Master}$ are equivalent as (3.9) and (3.11).

In the following we confirm that the dual equivalence persists at quantum level up to contact terms. If we define the generating function:

$$\mathcal{Z}(J) = \int \mathcal{D}A\mathcal{D}f\, e^{i\int d^3x\left[\mathcal{L}_{Master}^{New}(f,A)+\frac{\lambda(\partial_\mu A^\mu)^2}{2}+J^\alpha\tilde{A}_\alpha\right]} \tag{3.13}$$

After $A_\mu \to A_\mu + f_\mu$ and integrating over A^μ we obtain, up to an overall constant,

$$\mathcal{Z}(J) = \int \mathcal{D}f\, e^{i\int d^3x\left[\mathcal{L}_{GSD}(f)+J^\alpha D_{\alpha\beta}J^\beta+J^\alpha\tilde{f}_\alpha\right]} \tag{3.14}$$

with

$$D_{\alpha\beta} = \frac{-1}{4a_1a_0^2}\left[\left[(a_2^2\Box - a_1^2)E_{\alpha\beta} - 2a_1a_2\,\Box\,\theta_{\alpha\beta}\right]\right. \tag{3.15}$$

On the other side, if we integrate over f_μ in (3.13) we have $\mathcal{L}_{NL-HD-MCS}+\frac{\lambda(\partial_\mu A^\mu)^2}{2}+J^\alpha\tilde{A}_\alpha$. Therefore, deriving with respect to J_α we deduce:

$$\left\langle\tilde{f}_\alpha(x)\tilde{f}_\beta(y)\right\rangle_{GSD} = \left\langle\tilde{A}_\alpha(x)\tilde{A}_\beta(y)\right\rangle_{NL-MCS} - D_{\alpha\beta}\delta^{(3)}(x-y) \tag{3.16}$$

Furthermore, if we define the generating function

$$\mathcal{Z}_{GSD}(J,j) = \int \mathcal{D}f\, e^{i\int d^3x\left(\mathcal{L}_{GSD}(f)+J^\alpha\tilde{f}_\alpha+j^\alpha f_\alpha\right)} \tag{3.17}$$

and make $f_\mu \to f_\mu + J_\mu/2a_0$ we have

$$\mathcal{Z}_{GSD}(J,j) = \int \mathcal{D}f\,\exp\left\{i\int d^3x\left[\mathcal{L}_{GSD}(f) + (j^\alpha + J^\alpha)f_\alpha - \frac{J_\nu j^\nu}{2a_0} - \frac{J_\mu H^{\mu\beta}J_\beta + J_\nu J^\nu}{4a_0}\right]\right\} \tag{3.18}$$

where $H_{\alpha\beta}$ is the operator on the right handed side of (2.2), i.e., $\tilde{f}_\alpha \equiv H_{\alpha\beta}f^\beta$. From (3.17) and (3.18), see also [24, 18], we have

$$\left\langle f_\alpha(x)\tilde{f}_\beta(y)\right\rangle_{GSD} = \langle f_\alpha(x)f_\beta(y)\rangle_{GSD} + \frac{g_{\alpha\beta}}{2a_0}\delta^{(3)}(x-y) \tag{3.19}$$

$$\left\langle\tilde{f}_\alpha(x)\tilde{f}_\beta(y)\right\rangle_{GSD} = \langle f_\alpha(x)f_\beta(y)\rangle_{GSD} + \frac{(g_{\alpha\beta} + H_{\alpha\beta})}{2a_0}\delta^{(3)}(x-y) \tag{3.20}$$

Since the GSD model is quadratic the relations (3.19) and (3.20) assure that the classical self-duality $f_\mu = \tilde{f}_\mu$ holds also at quantum level up to contact terms. Combining (3.16) and (3.20) we derive:

$$\langle f_\alpha(x) f_\beta(y)\rangle_{GSD} = \left\langle \tilde{A}_\alpha(x)\tilde{A}_\beta(y)\right\rangle_{NL-MCS} - \left(D_{\alpha\beta} - \frac{(g_{\alpha\beta}+H_{\alpha\beta})}{2a_0}\right)\delta^{(3)}(x-y) \quad (3.21)$$

We conclude that $\mathcal{L}_{NL-MCS}(A)$ is dual to $\mathcal{L}_{GSD}(f)$ at classical and quantum level (up to contact terms) with the map $f_\mu \leftrightarrow \tilde{A}_\mu$. From this point of view $\mathcal{L}_{NL-MCS}(A)$ is on the same footing of $\mathcal{L}_{HD-MCS}(A)$ but it has the advantage of having the same spectrum of the GSD model and being ghost free.

We end up this section with three remarks. First, it is tempting to formally redefine the gauge field $A_\mu \to (a_0 - a_2\Box)^{1/2} A_\mu$ in order to make $\mathcal{L}_{NL-MCS}(A)$ a local gauge theory. However, the higher derivative theory thus obtained, although classically equivalent to the GSD model and simpler than $\mathcal{L}_{HD-MCS}$, contains two poles $k^2 = m_\pm^2$ and one of them is necessarily a ghost now. This is not totally surprising, the situation is similar to the massless Schwinger model in $1+1$ dimensions where the integration over the fermionic massless fields gives rise to the effective (exact) gauge invariant action:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{e^2}{4\pi}F^{\mu\nu}\frac{1}{\Box}F_{\mu\nu} \quad (3.22)$$

The photon propagator from the above lagrangian contains only one massive pole at $k^2 = e^2/\pi$ and no ghosts. However, if we try a naive redefinition $A_\mu \to \sqrt{\pm\Box}A_\mu$ the action becomes local but the propagator will get a factor $\pm\left[k^2(k^2 - e^2/\pi)\right]^{-1}$. Whatever sign we choose we always have a ghost field either at $k^2 = 0$ or $k^2 = e^2/\pi$. In fact in this specific case we can achieve a consistent local formulation by introducing a scalar field through $A_\mu = (\epsilon_{\mu\nu}\partial^\nu/\sqrt{\Box})\phi$. However, a consistent formulation in terms of a vector gauge field can only be non-local to the best we know.

The second remark concerns the uniqueness of the new master action. Clearly, the important point in our proposal is that the gauge and the non-gauge fields are mixed though a non-propagating term, in this case of $d = 2+1$ we have used the Chern-Simons term. One might try to generalize the new master action by using an arbitrary constant in front of the mixed Chern-Simons term in (3.4) instead of a_1. We have done that and checked that all important conclusions about dual equivalence, spectrum match, absence of ghosts, etc, hold for arbitrary values of this parameter but the simplest dual gauge theory is obtained precisely for the theory suggested here.

At last, we remark that the duality between $\mathcal{L}_{NL-MCS}(A)$ and the GSD model obtained by the new master action is also valid for the case where $a_0 = a_0(\Box)$, $a_2 = a_2(\Box)$ are not constants. If the coefficient of the Chern-Simons mixing term a_1 is kept constant we expect once again a spectrum equivalence otherwise the match of the spectrum between the non-gauge and the gauge theory will no longer be true in general.

4. Conclusion

Recently, the Noether gauge embedding and the master action procedures were applied in

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different theories and different dimensions. In particular, one has been trying to generalize them to nonabelian theories [25] and models defined on the non-commutative plane [26]. This embedding procedure usually reproduces the result of a master action which is very convenient at quantum level. Here we have shown how we can modify this master action to avoid extra poles in the propagator and assure that both dual theories have the same spectrum. Though, we have used a generalized self-dual model as an example, it is clear that similar ideas can be useful whenever we make use of interpolating master actions. The key point is to choose a non-propagating term to mix the gauge and non-gauge fields. The price we have paid for a ghost free dual gauge theory was locality. However, it is well know that it is not easy to have massive vector fields and still keep local gauge invariance, sometimes we have to break locality as in the massless Schwinger model. Since the GSD model contains two massive vector modes, one might speculate that the usual topological mechanism [27] to generate mass for a gauge field in $d = 2 + 1$, with the help of higher derivatives, could not cope with two massive poles without generating unwanted extra poles. More work is necessary to clarify this point.

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